Université de Monastir

Faculté des sciences de Monastir

On the Interfaces of Number Theory and Special Functions with Computer Algebra

Octobre 29 - 31, 2024

Monastir, Tunisia

Conference Chairs:

M. P. Chaudhary - *India and* **K. Mazhouda** - *Tunisia*

Program (at a glance)

Time	Duration	Tuesday	Wednesday	Thursday
8h-8h45		**Opening**		
8h45-9h30	45	Chaudhary	Sodaïgui	Hizem
9h35–10h20	45	Bhowmik	Kurşungöz	Taous
10h25–11h		**Coffee Break**	**Coffee Break**	**Coffee Break**
11h–11h45	45	Sekatskii	Darlison	Chems-Eddin
11h50–12h30	40	Baccar	Alanazi	Yucihiro
12h30–14h30		**Lunch**	**Lunch**	Hbaïb
				Lunch (13h-14h30)
14h30–14h55	25	Dabbabi		Meselem
15h–15h25	25	Khlifi		Guesmi
15h30–15h55		Mittou		Khmiri
16h–16h30		**Coffee Break**		**Coffee Break**
16h30–16h55	25	Milles		Elkhiri
17h-17h25	25	Guiben		Maatoug

Program (in details)

Program
Day one: Tuesday, 29 october, 2024

8h–8h45 Opening

Chair: Sodaïgui

8h45–9h30 On legacy of Indian mathematicians since ancient to modern era and contributions of Srinivasa Ramanujan

M. P. Chaudhary (International Scientific Research and Welfare Organization, India; Netaji Subhas University of Technology, **India**)

9h35–10h20 Goldbach and Riemann : some links

Gautami Bhowmik (Université de Lille, **France**)

10h25–11h Coffee Break

Chair: Chaudhary

11h–11h45 The generalized Littlewood theorem concerning integrals of analytical functions, and its use for analysis of zeroes of analytical functions

Sergey Sekatskii (University of Lausanne, **Switzerland**)

11h50–12h30 Sets with Even Partition Functions and Cyclotomic Numbers

Naceur Baccar (ISITCOM, University of Sousse, **Tunisia**)

Alternative Combinatorial Interpretations of

12h30–14h30 Lunch

Chair: Bhowmik

14h30–14h55 Les ensembles Fathomless et applications
Amir Dabbabi (University of Monastir, **Tunisia**)

15h–15h25 Several identities and relations related to q-analogues of Pochhammer k-symbol with applications to Fuss-Catalan-Qi numbers
Khlifi Mongia (University of Sfax, **Tunisia**)

15h30–15h55 Note on the coreful divisors
Brahim Mitou (University Kasdi Merbah Ouargla, **Algeria**)

16h–16h30 **Coffee Break**

Chair: Chems-Eddin

16h30–16h55 Particular Fuzzy Subsets on Topology Generated by Fuzzy Relation
Soheyb Milles (University Center of Barika,, **Algeria**)

17h–17h25 Les fonctions mock thêta et leurs applications
Salem Guiben (University of Monastir, Tunisia)

Program
Day two: Wednesday, 30 october, 2024

Chair: Ben Saïd

8h45–9h30 Steinitz classes of unramified Galois extensions
Bouchaib Sodaïgui (Université Polytechnique Hauts-de-France, **France**)

9h35–10h20 Cylindric partitions into distinct parts
Kağan Kurşungöz (Sabancı University, **Turkey**)

10h25–11h Coffee Break

Chair: Kurşungöz

11h–11h45 On Andrews-MacMahon theorem
Darlison Nyirenda (University of the Witwatersrand, **South Africa**)

11h50–12h30 Göllnitz-Gordon Identities and Little Göllnitz Identities
Abdulaziz Alanazi (University of Tabuk, **Saudi Arabia**)

13h–15h Lunch

Visit of Monastir Médina

Program
Day three: Thursday, 31 october, 2024

Chair: Sergey Sekatskii

8h45–9h30 Absorbing-factorizations in power series rings
Sana Hizem (University of Monastir, **Tunisia**)

9h35–10h20 On the Euclidean Ideals of the Ring of Integers of Certain Real Biquadratic Fields
Mohammed Taous (Mohammed First University, Oujda, **Morocco**)

10h25–11h Coffee Break

Chair: Hizem

11h–11h45 The Capitulation Kernel: techniques and characterizations
M. Chems-Eddin (Faculty of Sciences Dhar El Mahraz, Fès, **Morocco**)

11h50–12h20 Negative moments of Dirichlet L-functions
Yucihiro Toma (Nagoya University, **Japan**)

12h20–13h On the transcendance of quasi-periodic Rosen continued fractions
Mohamed Hbaïb (University of Sfax, **Tunisia**)

13h–15h Lunch

Chair: Taous

15h–15h25 Sum of an arithmetic function linked of the integer part
Meselem Karras (University of Tissemsilt, **Algeria**)

15h30–15h55 Rings with uniformly S-SF T
Guesmi Samir (University of Sousse, Tunisia)

16h–16h30 **Coffee Break**

16h30–17h
Superzeta functions of the second kind on function fields
Jawher Khmiri (University of Monastir, **Tunisia**)

Chair: Nyirenda

17h–17h25 Some super-congruences involving harmonic numbers and inverse binomial coefficients
Laid Elkhiri (Tiaret University, **Algeria**)

17h30–17h55 Mean values of some arithmetic functions under the hereditary sum of digits
Mabrouk Maatoug (University of Monastir, **Tunisia**)

List of Participants (Number of registers : 52)

Saib Abdessadek
Omar Adjali
Abdulaziz Alanazi
Bader Alqurashi
AYOUB Aluartassi
Sabrine Arfaoui
jilali Assim
SAFIA BATLA
Jihene Bel Hadj Ltaief
Houda Bellitir
Fethi Ben Said
ILHEM BENZAOUI
Gautami Bhowmik
Kajtaz Bllaca
M.P. Chaudhary
Mohamed Mahmoud Chems-Eddin
Abdelamir Dabbabi
Mohamed Dalah
Lamia Dammak
Ikrame Daqaq
Othman Echi
Taha EDDHAY
kaoutar El-Khalf
Salem guiben
Sana Hizem
manel jellali
Mazhouda Kamel
Meselem Karras
mohamed khalifa
Jawher Khmiri
Kagan Kursungoz
Ahmed Maatallah
Mabrouk Maatoug
Mohammed Mekkaoui
Mohammed MEZIANE
Soheyb Milles
Brahim Mittou
Hbaib Mohamed
Taieb Mohammed
Khlifi Mongia
Samir Moulahi
Iu-Iong Ng
Darlison Nyirenda
abdessalem riahi
BRUNDABAN SAHU
Guesmi Samir
Bouchaib SODAIGUI
Abdelhak Taane
Mohammed TAOUS
Yuichiro Toma
Amel Zergane
youcef نهاري محمد

Speakers and Abstracts

Conference Chairs :

M. P. Chaudhary (India)
and
K. Mazhouda (Tunisia)

Alternative Combinatorial Interpretations of Göllnitz-Gordon Identities and Little Göllnitz Identities

Abdalaziz Mohamed Alanazi

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Abstract

In this talk, we will describe new signed partition and overpartition interpretations for the first and second Göllnitz-Gordon identities, as well as for the first and second little Göllnitz identities. Both generating functions and bijective proofs will be provided.

Sets with Even Partition Functions and Cyclotomic Numbers

Naceur Baccar

ISITCOM, University of Sousse, Tunisia

Abstract

Let $P \in \mathbb{F}_2[z]$ be such that $P(0) = 1$ and degree $P \geq 1$. Nicolas et al proved that there exists a unique subset $A = A(P)$ of $\mathbb{N}$ such that

$$\sum_{n \geq 0} p(A, n) z^n \equiv P(z) \mod 2,$$

where $p(A, n)$ is the number of partitions of n with parts in A. Let m be an odd positive integer and let $\chi(A, \cdot)$ be the characteristic function of the set A. Finding the elements of the set A of the form $2^k m$, $k \geq 0$, is closely related to the 2-adic integer $S(A, m) = \chi(A, m) + 2\chi(A, 2m) + 4\chi(A, 4m) + \cdots = \sum_{k=0}^{\infty} 2^k \chi(A, 2^k m)$, which has been shown to be an algebraic number. Let G_m be the minimal polynomial of $S(A, m)$. In precedent works there were treated the case P irreducible of odd prime order p. Taking $p = 1 + ef$, where f is the order of 2 modulo p, explicit determinations of the coefficients of G_m have been made for $e = 2$, 3 and 4.

Goldbach and Riemann : some links

Gautami Bhowmik

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Abstract

We will deal relations between two classical problems of number theory : the Goldbach conjecture and the Riemann hypothesis. In particular we will consider the Hardy-Littlewood hypothesis on the problem Goldbach which rule out certain possible counterexamples of the GRH.

The Capitulation Kernel: techniques and characterizations

Mohamed Mahmoud CHEMS-EDDIN

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Abstract

Let L/K be an unramified cyclic extension of number fields. This talk consists of two parts. Firstly, we shall expose some useful techniques, for computing the capitulation kernel of L/K, that are mainly based on the computation of the unit group of L (resp. K) and a result of Heider-Schmithals, whenever L is a multiquadratic field. Secondly, we shall consider a commutative ring $(R, +, \times)$ and a subset A of R such that $(A, \times)$ is an abelian group. For any 1-cocycle $f : G \longrightarrow A$, we introduce the **Kummer resolvent** of f denoted (f, G) and defined by $(f, G) := \sum_{\sigma \in G} f(\sigma)$. By means of these Kummer resolvents, we give a characterization of the capitulation kernel for a class of extensions of number fields L/K.

Keywords

Capitulation Kernel, Unit Group, Kummer Resolvent.

References

[1] M. M. Chems-Eddin, Arithmetic of some real triquadratic fields; Units and 2-class groups, Moroc. J. Algebra Geom. Appl., 2024 (Article in press).

[2] M. M. Chems-Eddin, Unit groups of some multiquadratic number fields and 2-class groups, Period. Math. Hung., 84 (2022), 235-249.

[3] M. M. Chems-Eddin, A. Derhem, M. Talbi, Kummer resolvents and 3-class field tower of cyclic cubic fields, 2023 (Preprint).

[4] M. M. Chems-Eddin, M. B. T. El Hamam and M. A. Hajjami, On the unit group and the 2-class number of $\mathbb{Q}(\sqrt{2}, \sqrt{p}, \sqrt{q})$, Ramanujan J., (2024). https://doi.org/10.1007/s11139-024-00947-x

[5] F. P. Heider, B. Schmithals, Zur kapitulation der idealklassen in unverzweigten primzyklischen erweiterungen, J. Reine Angew. Math. 366 (1982), 1-25.

[6] K. Iwasawa, A note on the group of units of an algebraic number field. J. Math. Pures Appl., 35 (1956), 189-192.

On legacy of Indian mathematicians since ancient to modern era and contributions of Srinivasa Ramanujan

M. P. Chaudhary

International Scientific Research and Welfare Organization, India; Netaji Subhas University of Technology, India; Anand International College of Engineering, India; Mission Promote Research, India

Abstract

In this lecture we shall discuss about contributions of Indian mathematicians on various aspects of developments of mathematical sciences, since ancient to modern era. Furthermore, we shall see detailed contributions done by Ramanujan particularly in number theory special functions, and computer algebra.

References

- Invited Lecture ”Contribution of Indian Scholars in Mathematics, Science and Philosophy” Viewpoints 2007, Franklin Marshall College, Lancaster, USA, 11th June 2007.
- Plenary Address “Generalization of Character Formulas” 20th Internat. Math. Conf. of the Bangladesh Math. Soc., Bangladesh, December 8-10, 2017.
- Invited Lecture “Ramanujan’s Mathematics and its possible Applications to Applied Sciences” School of Biosciences and Engineering, Jadavpur University, Kolkata, India, 21st December 2017.
- Invited Talk “Ramanujan’s Second Last Discovery, Further Advancements and Generalizations” Centre for Mathematical, Computational and Data Science, Indian Association for the Cultivation of Science, Kolkata, India, 6th June 2018.
- DE068559531 George E. Andrews and Bruce C. Berndt, Ramanujan’s lost notebook, Part V., Cham: Springer, (2018). Zbl 1416.11001

Some super-congruences involving harmonic numbers and inverse binomial coefficients

Laid Elkhiri

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Abstract

In this presentation, we study some congruences modulo a power of a prime number p (super-congruences) in the ring of p-integers concerning sums involving inverse binomial and generalized harmonic numbers, for example we will

present some an intersting identities and congruences sush

$$\sum_{k=m}^{p-1} k^s \binom{p-1}{k}^n H_{k,m} \pmod{p^2}\ ,\ \sum_{k=m}^{\frac{p-1}{2}} \binom{p-1}{m}^n H_{k,m} \pmod{p^2},$$

$$\text{and} \quad \sum_{k=0}^{\left[\frac{p-1}{3}\right]} \binom{p-1}{k}^n H_{k,m} \pmod{p}.$$

Keywords

Harmonic numbers, binomial coefficients, congruences.

References

- L. Elkhiri, M. Mihoubi and A. Derbel, ”Congruences involving sums of harmonic numbers and binomial coefficients”, *Mathematica Montisnigri*, 26, (2020).
- E. Elkhiri, M. Mihoubi and A. Derbel, ”Congruences involving alternting sums related to harmonic numbers and binomial coefficeints”, *NNTDM journal*, 47, 39-52, (2020).
- L. Elkhiri, S. Koparal and N. Ömür, ”Congruences with q-harmonic numbers and q-binomial coefficients”, *Indian Journal of Pure and Applied Mathematics*, (2023).
- S. Koparal, L. Elkhiri and N. Ömür, ”On Congruences with Binomial Coefficients and Harmonic Numbers”, *Filomat*, 36, 3, 951-960, (2022).

Rings with uniformly S-SFT

Samir Guesmi

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Abstract

In this talk, we examine the notion of uniformly S-SFT and study its properties. Let R be a commutative ring and S a multiplicative subset of R. A ring R is said to be uniformly S-SFT if there exists an element s in S such that for every ideal I of R, there exist a finitely generated sub-ideal J of I and a positive integer n with the property that $sa^n \in J$ for all a in I. Our investigation includes proving Cohen's Theorem for uniformly S-SFT rings and analyzing the behavior of uniformly S-SFT property under various ring operations like Nagata's idealization and amalgamation of algebras.

Keywords

SFT ideal, Uniformly S-SFT ring, SFT ring, Cohen's Theorem, S-root.

References

- C. Bakkari, " Armendariz and SFT Properties in Subring Retracts," *Mediterranean J. math.*, vol. 6, pp. 339-345, (2009).
- S. Guesmi and A. Hamed, " Noetherian spectrum condition and the ring $\mathcal{A}[X]$," *J. Algebra Appl.*, (to appear).
- K. Louartiti and N. Mahdou, "Amalgamated algebra extensions defined by von neumann regular and SFT conditions," *Gulf J. math.*, vol. 1 pp. 105-113, (2013).

Les Fonctions mock thêta et leurs applications

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Abstract

Le but de mes travaux repose sur la démonstration combinatoire de conjecture sur les fonctions mock thêta. On commence par donner quelques relations entre les fonctions mock thêta et les fonctions combinatoires des partitions, Ainsi que des relations entre les fonctions mock thêta et les q-fractions continues et je termine par donner une idée sur la possible preuve combinatoire des conjectures sur les fonctions mock thêta.

Keywords

Mock thêta, combinatoire, q-fraction continue.

References

- M. P. Chaudhary, S. Guiben et K. Mazhouda, *Relations between mock Theta functions and combinatorial partition identities*, J. of Ramanujan Society of Mathematics and Mathematical Sciences Vol. 10, No. 1 (2022), pp. xx-yy DOI: 10.56827/JRSMMS.2022.1802.xx
- S. Guiben, *Relationships Between Mock Theta Functions and q-Continued Fractions.*, J. of Ramanujan Society of Mathematics and Mathematical Sciences Vol. 10, No. 2 (2023), pp. xx-yy DOI: 10.56827/JRSMMS.2023.1002.xx
- M. P. Chaudhary, S. Guiben et K. Mazhouda, *Combinatorial Proof of Mock Theta Conjectures*, Palestine Journal of Mathematics ©Palestine Polytechnic University-PPU 2024 Vol 13(2)(2024) , 285–295

On the transcendance of quasi-periodic Rosen continued fractions

Mohamed Hbaïb

University of Sfax, Faculty of sciences of Sfax, Tunisia

Abstract

In this talk, we consider the two Hecke groups G4 and G6 and we use the Schmidt Subspace Theorem to establish the transcendence of some quasi-periodic Rosen continued fractions in order to get the exact analogues of the results established with the regular continued fractions.

Keywords

References

- M. Hbaïb and M. Jellali, , Arabian Journal of Mathematics 13 (03) (2024).

Absorbing-factorizations in power series rings

Sana Hizem

University of Monastir, Faculty of sciences of Monastir, Tunisia

Abstract

Let R be a commutative ring with identity. A proper ideal I of R is said to be 1-absorbing prime ideal if whenever $xyz \in I$ for some nonunit elements $x, y, z \in R$, then either $xy \in I$ or $z \in I$. The ring R is called a 1-absorbing prime factorization ring (OAF-ring) if every proper ideal has an OA-factorization.

The ideal I of R is said to be 2-absorbing if whenever $xyz \in I$ for some elements $x, y, z \in R$, then either $xy \in I$ or $xz \in I$ or $yz \in I$. The ring R is called a two-absorbing factorization ring (TAF-ring) if every proper ideal of R has a two absorbing-factorization.

In this talk, we characterize commutative rings R (respectively, commutative ring extensions $A \subset B$) for which the ring of formal power series $R[[X]]$ (respectively, the ring $A + XB[[X]]$) is a OAF-ring or a TAF-ring.

Sum of an arithmetic function linked of the integer part

Meselem Karras

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Abstract

Let f be an arithmetic function such that $f(n) = O(n^{\varepsilon})$ for all positive ε and let $\lfloor . \rfloor$ denotes the integer part function. We give an asymptotic formula of the sum

$$\sum_{n_1 n_2 \ldots n_r \leq x} f(\lfloor \frac{x}{n_1 n_2 \ldots n_r} \rfloor).$$

Keywords

Divisor function, integer part, Asymptotic formula.

References

- O. Bordellès, L. Dai, R. Heyman, H. Pan, I.E. Shparlinski, *On a sum of involving the Euler funtion* , J. Number Theory **202**(2019), 278-297.
- H. Iwanice and C. J. Mozzochi, *On the divisor and circle problems*, J. Number Theory **29**(1988), 60-93.
- K. Liu, J. Wu, and Z.S. Yang, *A variant of the prime number theorem*, Indag. Math. (N.S.) **33**(2022), 388-396.
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- J. Ma and H.Y. Sun, *On a sum of involving the divisor funtion*, Periodica Mathematica Hungarica. **83**(2021), 185-191.
- J. Ma and J. Wu, *On a sum involving the von Mangoldt function*, Periodica Mathematica Hungarica. **83**(2021), 39-48.
- G. Voronoi, *Surune function transcendante et ses applications a*

- *J. Stucky, The fractional sum of small arithmetic functions, J. Number Theorey,* **238**, *(2022), 731-739*
- *W.G. Zhai, On a sum of involving the Euler funtion, J. Number Theory* **211***(2020), 199-219.*

Superzeta functions of the second kind on function fields

Jawher Khmiri

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Abstract

We study the superzeta functions of the second kind on function fields. Furthermore, we give special values of those functions, relate them to the Li coefficients, deduce some interesting summation formulas.

Keywords

Function fields; Superzeta functions; Riemann hypothesis; Li coefficients.

References

- K. H. Bllaca, J. Khmiri, K. Mazhouda, B. Sodaïgui, *Superzeta functions on function fields*, Finite Fields Appl. 95, Paper No. 102367, 22 pp, (2024).
- K. H. Bllaca, K. Mazhouda, *Explicit formula on function fields and application: Li coefficients*, Ann. Mat. Pura Appl. (4) 200, no. 5, 1859–1869, (2021).
- K. Mazhouda, L. Smajlović, *Evaluation of the Li coefficients on function fields and applications*, Eur. J. Math. 5, no. 2, 540–550 (2019).
- A. Voros, *Zeta functions over zeros of zeta functions*, Lecture Notes of UMI 8, Springer, 2010.

- A. Weil, *Sur les courbes algébriques et les variétés qui s'en déduisent*, Publ. Inst. Math. Univ. Strasbourg, 7 (1945). Actualités Scientifiques et Industrielles [Current Scientific and Industrial Topics], No. 1041 Hermann & Cie, Paris, 1948. iv+85 pp.

Cylindric partitions into distinct parts

Kağan Kurşungöz
(joint work with Halime Ömrüuzun Seyrek)

Faculty of Engineering and Natural Sciences, Sabancı University Orta Mh. Üniversite Cd. No:27 Orhanlı Tuzla, 34956, Istanbul

Abstract

We will describe an alternative way to describe cylindric partitions. Then, we will present a method for constructing generating functions of cylindric partitions, and give some examples. Except for the need for the fundamental theorem of algebra at a step, the approach is algorithmic. This is joint work with Halime Ömrüuzun Seyrek; in particular, the fourth chapter of the preprint at https://arxiv.org/pdf/2308.14514 .

Mean values of some arithmetic functions under the hereditary sum of digits

Mabrouk Maatoug

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Abstract

The average mean of the Möbius function or the Euler totient function is of great interest in analytic number theory [1, 4, 5]. Techniques from analytic number theory, such as Dirichlet series and complex analysis, are often employed to

study these averages [2]. The study of averages of multiplicative functions often involves understanding their asymptotic behavior, determining explicit formulas or estimates for these averages, and exploring connections with other areas of mathematics, such as harmonic analysis and probability theory . The aim of this talk is to conduct a survey on some selected multiplicative arithmetic functions for large values of integers, while considering constraints imposed by the hereditary sum of digits function in base b [3], denoted by w_b. This investigation aims to provide insights into the behavior of these functions within the scope of the hereditary sum of digits.

References

[1] Delange H, Sur les fonctions q-additives ou q-multiplicatives, Acta Arithmetica 21 (1972) 285-298.

[2] Drmota M, Mauduit C and Rivat J, Primes with an average sum of digits, Compositio Mathematica 145 (2009) 271-292.

[3] C. Sanna, On the exponential sum with the sum of digits of hereditary base b notation, Integers, 14(A36) (2014) 1-10.

[4] J.E. van Lint and H.E. Richert. On primes in arithmetic progressions. Acta Arith., 11:209216, 1965.

[5] N. Wiener. A note on Tauberian theorems. Ann. of Math. (2), 33(4):787, 1932.

Particular Fuzzy Subsets on Topology Generated by Fuzzy Relation

Soheyb Milles

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Abstract

Recently, Mishra and Srivastava have introduced and studied the notion of fuzzy topology generated by fuzzy relation and some basic properties were proved. In this paper, we mainly investigate the lattice structure of fuzzy open sets in this topology and we show its various properties and characteristics. Additionally, we introduce two particular subsets on this lattice, the fuzzy ideal and the fuzzy filter. For each of these subsets, we fully characterize them in terms of its meet and its join operations.

Keywords

Fuzzy set; Lattice; Ideal; Filter; Topology.

References

- Mishra, S., Srivastava, R., "Fuzzy topologies generated by fuzzy relations," *Soft Computing*, vol. 22, pp. 373-385, 2018.
- Milles, S., Zedam, L. and Ewa, R., "Characterizations of intuitionistic fuzzy ideals and filters based on lattice operations," *J. Fuzzy Set Valued Anal*, vol. 17, pp. 143-159, 2017.
- Zadeh, L.A., "Fuzzy sets," *Information and Control*, vol. 8, pp. 331-352, 1965.
- Zadeh, L.A., "Similarity relations and fuzzy orderings," *Information Sciences*, vol. 3, pp. 177-200, 1971.
- Zedam, L., Milles, S., Bennoui, A., "Ideals and Filters on a Lattice in Neutrosophic Setting," *Applications and Applied Mathematics*, vol. 16, no. 2, pp. 1141–1154, 2021.

Note on the coreful divisors

Brahim Mittou

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Abstract

Hardy and Subbarao [1] defined a coreful divisor (or c-divisor) d of a positive integer n as a divisor with the same core as n, which means with the same set of distinct prime factors as n. The number of such divisors of n is denoted by $\tau^{(c)}(n)$, by convention, 1 is coreful divisor of itself so that $\tau^{(c)}(1) = 1$. In this note we give the following results:

$$\sum_{n\leq x} \tau^{(c)}(n) = \frac{\zeta(2)\zeta(3)}{\zeta(6)} x + O\left(x^{\frac{1}{2}}\right),$$

and

$$\limsup_{n\to\infty} \frac{\log \tau^{(c)}(n) \log\log n}{\log n} = \frac{\log 3}{3}.$$

Keywords

Arithmetic function, coreful divisor, maximal order.

References

1. G. E. Hardy and M. V. Subbarao, "Highly powerful numbers", *Congr. Numer.*, vol. 37, pp. 277-307, 1983.

Several identities and relations related to q-analogues of Pochhammer k-symbol with applications to Fuss-Catalan-Qi numbers

Khlifi Mongia

Department of Mathematics, Faculty of Sciences of Sfax, Sfax University, Sfax, Tunisia, Sfax University, khlifimonjia@gmail.com

Abstract

In this work, we establish several identities and relations involving q-analogues of the Pochhammer k-symbol. Moreover, we generalize several identities and relations for q-analogues of the Catalan numbers and the Catalan-Qi numbers.

Keywords

Pochhamme k-symbol; Catalan number; Catalan-Qi-number; Basic hypergeometric series; Gamma function.

References

- W. Chammam, "Several formulas and identities related to Catalan-Qi and q-Catalan-Qi numbers," *Indian J. Pure Appl. Math*, vol. 50(4), 1039-1048, 2019.
- F.Qi, X.-T. Shi and P. Cerone, " A unified generalization of the Catalan, Fuss, and Fuss-Catalan numbers," *Math. Comput. Appl* vol. 24(2), no. 16, 2019.

On Andrews-MacMahon theorem

Darlison Nyirenda

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Abstract

For a positive integer r, George Andrews proved that the set of partitions of n in which odd multiplicities are at least $2r + 1$ is equinumerous with the set of partitions of n in which odd parts are congruent to $2r + 1$ modulo $4r + 2$.

This was given as an extension of MacMahon's theorem ($r = 1$). In this talk, we present a simple extension of this theorem and study various combinatorial consequences.

Keywords

Partition; bijection; generating function
MSC 2010: 05A15, 05A17, 05A19, 05A30, 11P81.

The generalized Littlewood theorem concerning integrals of analytical functions, and its use for analysis of zeroes of analytical functions

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Abstract

Recently, we have established and used the generalized Littlewood theorem concerning contour integrals of the logarithm of analytical functions, see e.g. [1 - 7]. Briefly, this Theorem is the following statement connecting the value of the contour integral $\int_C F(z)g(z)\,\mathrm{d}z$ ($z = x + iy$) of meromorphic functions $g(z)$ and $F(z) = ln(f(z))$ with the location of zeroes $X_\rho^0 + iY_\rho^0$ and poles $X_\rho^{pole} + iY_\rho^{pole}$ of $f(z)$, and residues of $g(z)$ lying inside the contour C, which is a rectangle bounded by the lines $x = X_1$, $x = X_2$, $y = Y_1$, $y = Y_2$:

$$\int_C F(z)g(z)\,\mathrm{d}z = 2\pi i\left(\sum_{\rho_g}\mathrm{res}(g(\rho_g)\cdot F(\rho_g)) - \sum_{\rho_f^0}\int_{X_1+iY_\rho^0}^{X_\rho^0+iY_\rho^0} g(z)\,\mathrm{d}z + \sum_{\rho_f^{pol}}\int_{X_1+iY_\rho^{pol}}^{X_\rho^{pol}+iY_\rho^{pol}} g(z)\,\mathrm{d}z\right).$$

This theorem has close connection with the similarly named Littlewood theorem [8, 9], and the proofs of these theorems are also similar; for all necessary technical details see [1, 2].

If asymptotic of the product $g(z)F(z)$ is such that the contour integral value

tends to zero in the limit of infinitely large contours C, we obtain

$$\sum_{\rho_f^0} \int_{-\infty+iY_\rho^0}^{X_\rho^0+iY_\rho^0} g(z)\,\mathrm{d}z - \sum_{\rho_f^{pol}} \int_{-\infty+iY_\rho^{pol}}^{X_\rho^{pol}+iY_\rho^{pol}} g(z)\,\mathrm{d}z = \sum_{\rho_g} \mathrm{res}(g(\rho_g)F(\rho_g)),$$

and this formula has numerous applications to find different infinite sums and to study zeroes of analytical functions.

Earlier, we apply this Theorem to prove the generalized Li's criterion equivalent to the Riemann hypothesis [3] (see [10, 11]) for original Li's criterion), and attempted to use the obtained results to test the Riemann hypothesis [5]. Recently, this work was highlighted in the entry of the Encyclopedia of Mathematics and its Applications [12].

Later on, we exploited this Theorem to study zeroes of polygamma-, incomplete gamma- and incomplete Riemann zeta-functions [6], as well as elliptical functions [7] and the Hurwitz zeta-function. These results, as well as some new findings, will be presented at the Conference.

For illustration, below we give a few examples.

1. Generalized Li's criterion equivalent to the Riemann hypothesis is the following Theorem [3]:

Theorem 1 (Generalized Li's criterion) *Riemann hypothesis is equivalent to the non-negativity of all derivatives* $\frac{1}{(n-1)!}\frac{d^n}{dz^n}((z-a)^{n-1}\ln(\xi(z)))\,|_{z=1-a}$ *for all non-negative integers* n *and any real* $a < 1/2$*; correspondingly, it is equivalent also to the non-positivity of all derivatives for all non-negative integers* n *and any real* $a > 1/2$.

Here $\xi(z)$ is the Riemann xi-function. The analogue of this Theorem for $a = 1/2$ is considered in [4].

2. The following property of zeroes of the digamma function holds [6].

Theorem 2 *Let* ρ_i *with* $i = 1, 2, 3\ldots$ *be real negative zeroes of digamma function* $\psi(z)$ *arranged in decreasing order, and* $\rho_0 = 1.461632\ldots$ *is the only one positive zero of this function. Then* $\lim_{N\to\infty}(\ln N + \sum_{n=0}^{N} \frac{1}{\rho_i}) = 0$.

For similar statements about zeroes of the polygamma functions, see [6].

3. Among numerous formulae describing the sums over inverse powers of zeroes of elliptic functions [7], we present the following. Let ρ_i be zeroes of the Weierstrass function $\wp(z,\tau)$. Then $\sum_{\rho_i} \frac{1}{(\rho_i^{zero})^4} = -10\delta_4$, $\sum_{\rho_i} \frac{1}{(\rho_i^{zero})^6} = -28\delta_6$, $\sum_{\rho_i} \frac{1}{(\rho_i^{zero})^8} = -54\delta_8 + 36\delta_4^2$, etc., where for $j>0$, $\delta_{2j}(\tau) = \sum_{n=-\infty}^{\infty} \sum_{\substack{m=-\infty \\ |m|+|n|\neq 0}}^{\infty} \frac{1}{(m+n\tau)^{2j}}$.

4. The following Theorem holds.

Theorem 3 *For an arbitrary large positive integer N and arbitrary small real ϵ, we can find such real value of $z_0(N,\epsilon)$ that the function $\zeta(s,z)$ with $|z| \leq z_0$ has at least N zeroes in the area $|s| < \epsilon$.*

Here $\zeta(s,z)$ is the Hurwitz zeta-function.

5. Finally, and more for the curiosity, we present the following result [6].

Let ρ_i be the roots of equation $f(z) = e^{bz} - a = 0$ having order k_i. Then for $a \neq 1$, $\sum_{\rho_i} \frac{k_i}{\rho_i^2} = \frac{b^2}{(1-a)^2} - \frac{b^2}{1-a}$. If $a = 1$ and $b \neq 0$, we have $\sum_{\rho_i \neq 0} \frac{1}{\rho_i^2} = -\frac{b^2}{12}$. This is simply the statement $\sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty} \frac{b^2}{(2\pi n i)^2} = -\frac{b^2}{12}$ (Basel problem solution; quite similarly, we can prove $\zeta(4) = \frac{\pi^4}{90}$, etc). Let now our equation be $f(z) := \exp(bz) - 1 - bz = 0$ with $b \neq 0$. Then $\sum_{\rho_i} \frac{1}{\rho_i^2} = -\frac{b^2}{18}$ (can this be named "the general Basel problem"?), etc.

Keywords: Generalized Littlewood theorem, logarithm of an analytical function, zeroes and poles of analytical function.
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Classes de Steinitz d'extensions galoisiennes non ramifiées (Steinitz classes of unramified Galois extensions)

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Résumé

Soient K un corps de nombres et Cl_K son groupe de classes. Soit Γ un groupe fini. On désigne par $R_{nr}(K,\Gamma)$ le sous-ensemble de Cl_K formé par les classes réalisables comme classes de Steinitz d'extensions galoisiennes de K, non ramifiées aux places finies de K et ayant un groupe de Galois isomorphe Ã Γ. Si Γ est abélien et le nombre de classes de K au sens restreint est premier avec l'ordre de Γ, alors $R_{nr}(K,\Gamma) = \emptyset$. Dans cet exposé, pour Γ quelconque on considère l'ensemble $R'_{nr}(K,\Gamma) := \{1\} \cup R_{nr}(K,\Gamma)$. On prouve que $R'_{nr}(K,\mathbb{Z}/2\mathbb{Z})$ est un sous-groupe de $Cl_{K,2} := \{c \in Cl_K | c^2 = 1\}$; de plus il est égal Ã $Cl_{K,2}$ sous une certaine hypothése sur K. En utilisant ce résultat, on montre que $R'_{nr}(K,\Gamma)$ est un sous-groupe de $R'_{nr}(K,\mathbb{Z}/2\mathbb{Z})$ si Γ est d'ordre impair, ou bien a un 2-sous-groupe de Sylow non cyclique (par exemple Γ un 2-groupe non abÃ©lien, $\Gamma = S_n$ ou A_n, avec $n \geq 4$), ou bien a un 2-sous-groupe de Sylow cyclique et normal (par exemple Γ nilpotent d'ordre pair).

Les résultats sont obtenus en collaboration avec Ayoub Al Uartassi (UPHF et Université Moulay Ismaïl, Meknès, Maroc) et Mohammed Taous (Université Mohammed Premier, Oujda, Maroc).

Abstract

Let K be a number field and Cl_K its class group. Let Γ be a finite group. We denote by $R_{nr}(K,\Gamma)$ the subset of Cl_K formed by the classes which are

realizable as Steinitz classes of Galois extensions of K, unramified at the finite places of K and having a Galois group isomorphic to Γ. If Γ is abelian and the narrow class number of K is prime to the order of Γ, then $R_{nr}(K,\Gamma) = \emptyset$. In the present talk, for any Γ we consider the set $R'_{nr}(K,\Gamma) := \{1\} \cup R_{nr}(K,\Gamma)$. We prove that $R'_{nr}(K,\mathbb{Z}/2\mathbb{Z})$ is a subgroup of $Cl_{K,2} := \{c \in Cl_K | c^2 = 1\}$; furthermore, it is equal to $Cl_{K,2}$ under a certain assumption on K. Using this result we show that $R'_{nr}(K,\Gamma)$ is a subgroup of $R'_{nr}(K,\mathbb{Z}/2\mathbb{Z})$ if Γ either has odd order, or has a noncyclic 2-Sylow subgroup (for instance Γ a nonabelian 2-group, $\Gamma = S_n$ or A_n, with $n \geq 4$), or has a normal cyclic 2-Sylow subgroup (for instance Γ nilpotent having even order).

The results are obtained in collaboration with Ayoub Al Uartassi (UPHF et Université Moulay Ismaïl, Meknès, Maroc) et Mohammed Taous (Université Mohammed Premier, Oujda, Maroc).

On the Euclidean Ideals of the Ring of Integers of Certain Real Biquadratic Fields

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Abstract

Lenstra [4] introduced the concept of Euclidean ideal classes, extending the notion of a Euclidean domain. He also demonstrated that the existence of a Euclidean ideal class implies that the class group is cyclic and generated by the Euclidean class. In this work, we develop new approach to prove the existence of a Euclidean ideal class in real biquadratic number fields with class numbers that are powers of two. This research generalizes the previous works of Hsu [2], Chattopadhyay and Muthukrishnan [1], and Krishnamoorthy and Pasupulati [3].

Keywords

Euclidean ideal class, cyclic class group, biquadratic field, class number.

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Negative moments of Dirichlet L-functions

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Abstract

Cryptographic schemes based on lattice problems are considered to be candidates for quantum computation-resistant cryptography. Some algebraic lattice-based cryptographic schemes rely on the hardness of computing short generators of a given principal ideal. We focus on the subproblem which is a fundamental problem in lattice-based cryptosystems called the Short Generator Problem (SGP) aiming to recover a sufficiently short generator for the principal ideal from a given generator of the ideal.

In EUROCRYPT2016, Cramer, Ducas, Peikert and Regev (CDPR16) proposed an efficient algorithm for recovering short generators of principal ideals in q-th cyclotomic fields with q being a prime power. In order to prove the correctness of the algorithm, CDPR16 showed that it suffices to estimate the negative square moment of Dirichlet L-functions $L(s,\chi)$ at $s=1$ for even characters.

In present talk, we estimate the negative square moment of Dirichlet L-functions at $s=1$, and obtain the following asymptotic formula.

Let q be a positive integer and χ be a Dirichlet character modulo q. Then we have

$$\sum_{\substack{\chi\neq\chi_0 \\ \chi(-1)=1}} \frac{1}{L(1,\chi)^2} = \frac{\zeta(2)}{2\zeta(4)} \prod_{p|q} \left(1-\frac{1}{p^2}\right)^{-1} \varphi(q) + O_{\varepsilon}\left(q^{\varepsilon}\right).$$

In the case q being a prime number, we give the optimal lower bound of success probability of the algorithm given by CDPR16. This is a joint work with Iu-Iong Ng (arXiv:2405.18778).

Keywords

negative moments of Dirichlet L-functions, Gonek conjecture, Short Generator Problem (SGP), log-cyclotomic-unit lattice.

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